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The effect on the Asian option price times between the averaging

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Abstract

The article refers to the calculation of the price of Asian option. In particular, we analyze the effect on the option price times between the averaging. Review shall be conducted for both Arithmetic and Geometric Mean in discrete and continuous averaging.

Payoff at expiration T for Asian Call option with Arithmetic Mean equals:

$$C = Max(A - K, 0)$$
(1)

With Geometric Mean:

$$C = Max(G - K, 0)$$
(1_1)

Where

A – Arithmetic Mean of S(t) – stock price at time t during interval T;
G – Geometric Mean of S(t) – stock price at time t during interval T;
K – Strike price.

In accordance with the Black-Sholes model price change stock S (t) is given by:

$$dS(t) = S(t)(r_d - r_f)dt + S(t)\sigma dW(t)$$
$$0 \le t \le T$$
(2)

Where

r_d = constant interest rate;
r_f = constant dividend rate;
σ = constant volatility.

W(t) – standard Winer process.

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That is:

$$S(t) = e^{x(t)}$$

(3)

In accordance with the Black-Sholes model the random variables x(t) form a normal, Gaussian stochastic process with the vector of the expectation of m (t) and covariance matrix Covmat (t1, t2), calculated according to the formulas:

$$m(t) = LnS_0 + (r_d - r_f - 0.5\sigma^2)t = LnS_0 + \gamma t$$

$$Covmat(t1,t2) = \sigma^2 Min(t1,t2)$$

(4)

Where

S_0 – initial stock price value.

Let A – arithmetic mean, G – geometric mean. In continuous case:

$$A = \frac{1}{T}\int_0^T S(\tau)d\tau = \frac{1}{T}\int_0^T e^{x(\tau)}d\tau$$

$$G = e^{\frac{1}{T}\int_0^T LnS(\tau)d\tau} = e^{\frac{1}{T}\int_0^T x(\tau)d\tau} = e^y$$

(5)

In discrete case:

$$A = \frac{1}{n}\sum_{i=1}^{n} S(i)$$

$$G = \left(\prod_{i=1}^{n} S(i)\right)^{\frac{1}{n}} = e^{\frac{1}{n}\sum_{i=1}^{n} LnS(i)} = e^{\frac{1}{n}\sum_{i=1}^{n} x(i)}$$

(6)

We calculate the first and second moments of the random variables A and G.

Random variable $y = \frac{1}{T}\int_0^T x(\tau)d\tau$ – gauss variable with math. expectation and dispersion equals:

$$Ey = \frac{1}{T}\int_0^T E(x(\tau))d\tau = \frac{1}{T}\int_0^T \gamma\tau d\tau = LnS_0 + \frac{\gamma T}{2}$$

$$\sigma_y^2 = \frac{1}{T^2}\int_0^T\int_0^T Cov(x(\tau),x(\eta))d\tau d\eta = \frac{1}{T^2}\int_0^T\int_0^T \sigma^2 Min(\tau,\eta)d\tau d\eta = \frac{\sigma^2 T}{3}$$

(7)

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Random variable $yd = \frac{1}{n}\sum_{i=1}^{n} x(i)$ – gauss variable with math. expectation and dispersion equals:

$$Eyd = \frac{1}{n}\sum_{i=1}^{n} m(i)$$

$$\sigma_{yd}^2 = \frac{1}{n^2}\left(\sum_{i=1}^{n} Covmat(i,i) + 2\sum_{i=1}^{n}\sum_{j>i}^{n} Covmat(i,j)\right)$$

(8)

In continuous case:

$$E(A) = \frac{1}{T}\int_0^T E(e^{x(\tau)})d\tau = \frac{S_0}{T}\int_0^T e^{(\gamma+0.5\sigma^2)\tau}d\tau = \frac{S_0}{T}\frac{e^{(rd-rf)T}-1}{rd-rf}$$

$$E(G) = e^{Ey+0.5\sigma_y^2}$$

$$E(A^2) = \frac{1}{T^2}\int_0^T\int_0^T E(e^{x(\tau)+x(\eta)})d\tau d\eta = \frac{1}{T^2}\int_0^T\int_0^T e^{2LnS_0+(rd-rf)(\tau+\eta)+\sigma^2\min(\tau,\eta)}d\tau d\eta =$$

$$= \frac{2S_0^2}{T^2}\frac{1}{rd-rf+\sigma^2}\left(\frac{e^{(2(rd-rf)+\sigma^2)T}-1}{2(rd-rf)+\sigma^2} - \frac{e^{(rd-rf)T}-1}{rd-rf}\right)$$

$$E(G^2) = e^{2(Ey+\sigma_y^2)}$$

(9)

In discrete case:

$$E(A) = \frac{1}{n}\sum_{i=1}^{n} e^{m(i)+0.5Covmat(i,i)}$$

$$E(A^2) = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} e^{m(i)+m(j)+0.5(Covmat(i,i)+Covmat(j,j)+2Covmat(i,j))}$$

$$E(G) = e^{Eyd+0.5_{yd}^2}$$

$$E(G^2) = e^{2(Eyd+\sigma_{yd}^2)}$$

(10)

The Geometric Mean G has log-normal distribution with parameters E_y, E_{yd} and σ_y, σ_y^d (see formulas (7) and (8)). For such a distribution option price can be calculated from the exact formula of Black-Scholes type:

$$\Pr C_G = e^{-rdT}\int_{Lnk}^{\infty}(e^y - K)e^{-\frac{1}{2\sigma_y^2}(x-m_x)^2}dx = e^{-rdT}\left(e^{m_y+0,5\sigma_y^2}Lp\left(\frac{\sigma_y^2+m_y-LnK}{\sigma_y}\right) - KLp\left(\frac{m_y-LnK}{\sigma_y}\right)\right)$$

$$Lp(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}u^2}du$$

(11)

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In discrete case:

$$m_y = E_{yd}$$
$$\sigma_y = \sigma_y^d$$
(12)

In continuous case:

$$m_y = E_y$$
$$\sigma_y = \sigma_y$$
(13)

For calculating the price of Asian option with Arithmetic Mean Levi and Turnbull [2] supposed that the Arithmetic Mean of the log. normal values itself has a log. normal distribution, i.e.

$$A = e^x$$
$$p(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{1}{2\sigma_A^2}(x-m_x)^2}$$
(14)

This distribution unknown parameters m_x, σ_x can be calculated using the following formula:

$$EA = e^{m_x + 0.5\sigma_x^2}$$
$$EA^2 = e^{2(m_x + \sigma_x^2)}$$

(15)

From (15) we get:

$$m_x = Ln\left(\frac{(EA)^2}{\sqrt{EA^2}}\right)$$
$$\sigma_x^2 = Ln\left(\frac{EA^2}{(EA)^2}\right)$$

(16)

According to Levi and Turnbull [2] price of Asian Call equals:

$$\Pr C_A = e^{-rdT}\int_{Lnk}^{\infty}(e^x - K)e^{-\frac{1}{2\sigma_x^2}(x-m_x)^2}dx = e^{-rdT}(e^{m_x+0,5\sigma_x^2}Lp(\frac{\sigma_x^2+m_x-LnK}{\sigma_x}) - KLp(\frac{m_x-LnK}{\sigma_x}))$$

$$Lp(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}e^{-\frac{1}{2}u^2}du$$

(17)

At a known price of the call option value of a put can be found from the relationship Put – Call parity:

$$\Pr P_A = \Pr C_A + (K - EA)e^{-rdT}$$
$$\Pr P_G = \Pr C_G + (K - EG)e^{-rdT}$$

(18)

The above formulas allow us to calculate option prices for the discrete and continuous case option with arithmetic and geometric averaging at different times of averaging. As an example, we examined the options with the parameters given in [1]. The chosen parameters are: S0 - initial asset price = 100, K-strike price = 100, rd-riskless rate of interest = 0.09, rf-dividend rate=0, sigma-volatility = 0.3, time between averaging points = 1d, 1week, 1month, 1 quarter, T-total averaging time = 1year.

The calculation results are shown in Table 1.

Table 1.

	Arithmetic			Geometric		
	Call	Put	Volatility	Call	Put	Volatility
Continuous	8.885756	4.646859	0.175809	8.323595	4.831282	0.173205
1day	8.882353	4.643217	0.17571	8.318312	4.827847	0.173091
1week	8.845788	4.617196	0.174879	8.272499	4.802663	0.172177
1month	8.708342	4.522292	0.171805	8.101823	4.709561	0.168782
1quarter	8.561474	4.362192	0.167477	7.85859	4.548671	0.163459

Parameter Volatility in Table 1 is calculated by formula:

$$Volatility = \frac{\sigma_y(\sigma_x)}{\sqrt{T}}$$

As can be seen from Table 1, the price of the option does not depend strongly on the time averaging. Nevertheless dependence occurs. The dependence is to some extent paradoxical. It would seem that an increase in the number of points averaging volatility should decrease, and the price of the option should grow. In fact, judging from the data of Table 1 the opposite thing is true: the more points the more averaging volatility. The explanation lies in the fact that the averaged random points are not independent. They are strongly correlated, what leads to the results. Nevertheless, even with the minimum two points averaging significantly reduces the price of the option. For comparison, we give the values of the prices of vanilla Call and Put with the same parameters. Vanilla Call price equals 16.21927, Put – 7.612387.

Reference.

[1] Michael Curran. Valuing Asian and Portfolio Options by Conditioning on the Geometric Mean Price. Portfolio Engineering. NOV-18-1993

[2] Levi,Edmond and Turnbill,Stuart,"Average Intelligence",Risk,February 1992.